NUMERICAL SIMULATION OF WEAK TURBULENT KOLMOGOROV SPECTRUM
IN WATER SURFACE WAVES

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1. INTRODUCTION

Gravity waves in water surface are characterized by a small steepness value. It makes possible to apply the weak-turbulence approach in order to find out the problem solution with the help of kinetic equation for energy spectrum. This equation was formulated by K.Hasselmann (1962,1963) and V.Zakharov (1968). In terms of wave action spectrum the equation is as follows:

$$\frac{\partial N(\boldsymbol{k})}{\partial t} = G_{nl} = \iiint T(\boldsymbol{k},\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3})\,\boldsymbol{\delta}(\boldsymbol{k}+\boldsymbol{k_1}-\boldsymbol{k_2}-\boldsymbol{k_3})\,\boldsymbol{\delta}(\omega+\omega_1-\omega_2-\omega_3)\times$$

$$\times\{N_2 N_3(N+N_1)-N_1 N(N_2+N_3)\}d\boldsymbol{k}_1 d\boldsymbol{k}_2 d\boldsymbol{k}_3 \tag{1}$$

where $N_i = N(\boldsymbol{k}_i)$ is a spectral density of wave action; $T(\boldsymbol{k},\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3})$ is a kernel function of non-linear wave interaction; $\boldsymbol{\delta}(\boldsymbol{k})$ and $\delta(\omega)$ are the delta-functions describing a resonance interaction between four wave components:

$$\boldsymbol{k}+\boldsymbol{k_1}=\boldsymbol{k_2}+\boldsymbol{k_3} \;; \tag{2a}$$

$$\omega+\omega_1=\omega_2+\omega_3 \tag{2b}$$

.

For gravity waves $\omega(k)=\sqrt{gk}$, the value $T(\boldsymbol{k},\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) \sim k^3$ is a homogenous function of the third order. An explicit expression for $T(\boldsymbol{k},\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3})$ can be found out in original papers (Hasselmann 1962, 1963; Webb 1978; Zakharov 1999).

The equation (1) has been a subject of numerical modeling for almost three decades (Webb 1978, K.Hasselmann 1985; Resio and Perrie 1991; Polnikov 1993; Komen et al., 1994; Komatsu and Masuda 1996, Zakharov 1999, Lavrenov 1998, 2001 etc.). However, some basic properties of this equation are not clarified till now.

Usually the equation (1) is considered to preserve standard constants of motion – wave action, energy and momentum:

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$$N = \int N(\boldsymbol{k})\, d\boldsymbol{k}$$

$$E = \int \boldsymbol{w}(\boldsymbol{k})\, N(\boldsymbol{k})\, d\boldsymbol{k} \tag{3}$$

$$\boldsymbol{K} = \int \boldsymbol{k}\, N(\boldsymbol{k})\, d\boldsymbol{k}$$

Actually, only N is a real constant of motion. The energy E and the momentum $\boldsymbol{K}$ are only "formal" integral of motion, "leaking" in the area of very large wave numbers. The high frequency truncation of the Hasselmann equation leads to a leakage of the wave energy and momentum to a high frequency range, whereas the wave action flux is mainly directed to a low frequency range. It can be presented as follows:

$$\frac{d\,N}{d\,t} = 0; \qquad \frac{d\,E}{d\,t} = -\,P; \qquad \frac{d\,\boldsymbol{K}}{d\,t} = -\boldsymbol{M}\,; \tag{4}$$

where P, $\boldsymbol{M}$ are fluxes of energy and momentum in $\boldsymbol{k}$-space directed to high frequency area.

Preservation of energy and momentum fluxes to high wave numbers is due to formation of weak turbulent Kolmogorov spectra. The main theoretical point of the kinetic equation (1) is a description of the stationary equation solution of :

$$G_{nl} \equiv 0 \tag{5}$$

The simplest Kolmogorov weak-turbulent stationary solution of the equation (1) was obtained in 1966 (Zakharov and Filonenko, 1966):

$$S(\omega) = \alpha_0 \frac{g^{4/3} P^{1/3}}{\omega^4} \tag{6}$$

where $S(\boldsymbol{w})$ is the energy spectrum defined by the relation:

$$S(\boldsymbol{w})\, d\boldsymbol{w}\, d\boldsymbol{b} = \boldsymbol{w}(\boldsymbol{k})\, N(\boldsymbol{k})\, d\boldsymbol{k} = \frac{2\boldsymbol{w}}{g^2} N(\boldsymbol{w}, \boldsymbol{b})\, d\boldsymbol{w}\, d\boldsymbol{b} \tag{7}$$

In general terms the Kolmogorov spectra are anisotropic:

$$S(\boldsymbol{w}, \boldsymbol{b}) = \frac{g^{4/3} P^{1/3}}{\boldsymbol{w}^4} F\!\left(\frac{g\,M}{\boldsymbol{w}P}, \boldsymbol{b} \right) \tag{8}$$

where $F = F(\boldsymbol{x}, \boldsymbol{b})$ is a function of two variables.

A solution (9) averaged over the angles is very close to energy frequency spectra, obtained in many laboratory and field experiments (Toba 1972; Donelan et al. 1985).

Now, a question arises, how can the exact Kolmogorov solution of the stationary equation (5) approximate solutions of the nonstationary equation (1), especially in cases of applying forcing and damping sources? It should be noted that for the first time an attempt to give an answer to this question is undertaken in paper (Komen, Hasselmann and Hasselmann,1984). They show that even in the presence of input and dissipation there is the spectrum solution $S \sim 1/\omega^4$. The nonlinear flux is found out to be generally rather strong, so that relatively small deviations from the solution $S \sim 1/\omega^4$ are sufficient to generate divergent fluxes, which can balance nonzero input and dissipation source function in the cascade region (Komen et al.,1994). Unfortunately, at that time it was difficult to get an exact estimation of basic parameters of problem solution.. A stationary solution obtained by solving nonstationary equation

(1) should be developed with reliable accuracy. That is why it is necessary to use sufficiently accurate and fast numerical algorithm, not available at that time.

It should be noted that a full-scale experiment on numerical simulation of the equation (1) is performed recently (Pushkarev et al., 2002). The approach, based on another numerical algorithm (Webb 1978; Resio&Perrie, 1991), existing for more than two decades, is used. As this problem seems to be rather complicated for numerical simulations and the results are obtained for the first time, there appears a necessity to produce independent estimations in order to verify the above-mentioned results with the help of the another algorithm.

This is a motivation for writing present paper. Numerical results of the equation (1) solution taking into account wave energy forcing and damping are obtained in this paper with the help of the numerical algorithm elaborated recently by one of the authors (Lavrenov 1998, 2001). The algorithm is based on numerical methods of highest precision. It is of high accuracy and calculation speed.

2. PROBLEM FORMULATION

The equation of spectral wave action evolution can be written as:

$$\frac{\partial N}{\partial t} + \boldsymbol{g} N = G_{nl} + F \qquad (9)$$

where G_{nl} is the non-linear energy transfer function (1); $\boldsymbol{g}$ is a damping increment depending on the frequency $\boldsymbol{w}$ as follows :

$$\boldsymbol{g} = \begin{cases} C_1 \boldsymbol{w}_{\min} \sin^2\left((1 - \boldsymbol{w}/\boldsymbol{w}_{\min})(\boldsymbol{p}/2)\right), & \text{if } \boldsymbol{w} < \boldsymbol{w}_{\min} \\ 0, & \text{if } \boldsymbol{w}_{\min} < \boldsymbol{w} < \boldsymbol{w}_f \\ C_2 \boldsymbol{w}_p \left(\boldsymbol{w}/\boldsymbol{w}_p - 1\right)^2, & \text{if } \boldsymbol{w} > \boldsymbol{w}_p \end{cases} \qquad (10),$$

where $C_{1,2}$ are positive constants.

The value F is an external active force: $F = f \, N$. The function $f = f(\omega, \beta)$ is a value not equal to zero within the frequency range: $\boldsymbol{w}_{\min} < \boldsymbol{w} < \boldsymbol{w}_f$. It is equal to the following angular function:

$$f = \begin{cases} Q\cos^n(A\boldsymbol{b}), \text{if} \quad \cos(A\boldsymbol{b}) > 0 \ \text{and} \ \boldsymbol{w}_{\min} < \boldsymbol{w} < \boldsymbol{w}_f \\ 0, \quad \text{in another cases} \end{cases} , \qquad (11)$$

where Q is a normalizing function, providing the same integral value for the various parameters n and A:

$$\int_{-\pi}^{\pi} f(\beta, \omega) d\beta = C_3 \omega_f \qquad (12)$$

where C_3 is a constant.

So, a problem is posed in such a way that the whole frequency range is divided into sub-ranges with different energy sources. The low-frequency damping domain is located in the range $(\boldsymbol{w} < \boldsymbol{w}_{\min})$ in order to stabilize fluxes directed into the low frequency range. The energy pumping domain is located within the range $(\boldsymbol{w}_{\min} < \boldsymbol{w} < \boldsymbol{w}_f)$. One of the most interesting frequency ranges is the domain $(\boldsymbol{w}_f < \boldsymbol{w} < \boldsymbol{w}_p)$ without any damping or pumping, i.e. it is the so-called "transparency window", where the spectrum is formed only by non-linear

energy transfer. The Kolmogorov constants should be defined using numerical results in this range. The high frequency damping is located in the domain ($W > W_p$). Values of the frequencies $\omega_{min}; \omega_f, \omega_p$ are defined in such a way that the appropriate frequency ranges include sufficient number of grid points to obtain reliable integral estimations defined by spectrum values within the corresponding frequency ranges. The following values are used:

$\omega_{min} = 0.5; \quad \omega_f = 1.0, \omega_p = 6.5$

In order to comply with the requirements of the kinetic equation (1) application, the coefficient C_3 in (12) is defined as $C_3 = 0.001$ satisfying the conditions of smallness of the growth rate with respect to the corresponding frequency: $f << W$. The coefficients $C_{1,2}$ as well as the appropriate frequencies $W_{min}; W_f, W_p$ are defined experimentally. The conditions of effectiveness of fluxes absorption and minimization of the pumping and damping intervals with respect to appropriate frequency are used. It should be noted that exact values of the constants $C_{1,2}$ (satisfying the conditions $C_{1,2} < 1.0$) are not so principal to obtain a qualitative solution within the range $W_{min} < W < W_p$. The values $C_{1,2}$ are defined in such a way that the energy, wave action and momentum fluxes are absorbed by dissipation within appropriate frequency ranges and by numerical accuracy of fluxes estimations.

Numerical results are estimated in 96 directions and 50 frequencies. Such a detailed angular resolution is used for obtaining an accurate estimation for narrow angular distributions of external force approximation (12) considered in paper. In this case the angular increment is equal to $\Delta\beta/2\pi \approx 0.065$, being the same order of frequency increment: $\Delta\omega/\omega \approx 0.068$. Such discritization is optimal for numerical integration in general case, when solution is not known before hand (Lavrenov, 1998).

As the solution is to be obtained for a large time scale (up to 100000 seconds), an optimal numerical algorithm of non-linear energy transfer computation is used (Lavrenov 1998, 2001).

3. NUMERICAL RESULTS

3.1 Spectrum evolution

Now, the numerical results for the isotropic case with $n = 0.0$ and $A=1.0$ should be considered. The frequency spectrum can be defined as a function of wave action as follows:

$$S(W) = \int_{-p}^{p} S(W, b) \, db = \int_{-p}^{p} N(W, b) \, 2W^4 / g^2 \, db \qquad (13)$$

The spectrum (13) for the following time steps: 100, 300, 500, 1000, 3000, 5000, 10000, 30000 and 50000 sec., respectively, is shown in the logarithmic scale (Fig 1).

Two different stages can be defined in the wave spectrum evolution. On the first stage the spectrum growth is observed within the range of the external force impact: $0.5 < w < 1.0$. The spectrum is quickly increased in more than 6 orders. Duration of this time interval is estimated as: $t \approx 10000$ sec. The spectrum becomes almost stationary at $t \geq 30000$ sec. Its values coincide exactly with the spectrum at $t \approx 50000$ sec. The initial and final stages of spectral evolution are presented in Fig. 1a. Within the range $0.8 < w < 8.0$, the stationary spectrum is very close to the Zakharov-Filonenko one:

$$S(w) \cong w^{-4.0 \pm 0.1} \qquad (14)$$

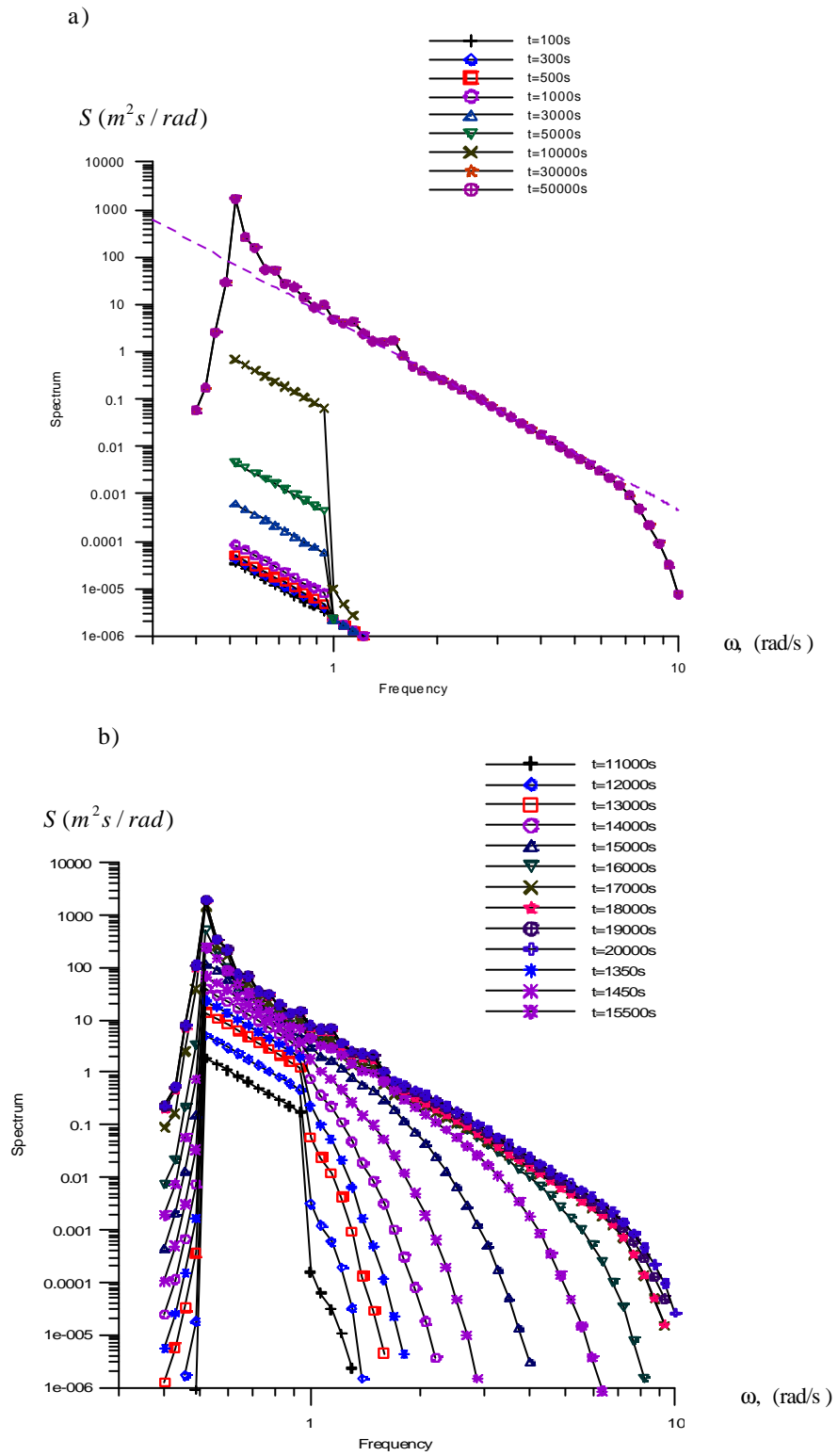Spectra evolution within the intermediate time interval $11000 < t < 15000$ sec. is presented in Fig.1b.

a)

$S\,(m^2 s\,/\,rad)$

Spectrum

$\omega,\ (rad/s)$

Frequency

b)

$S\,(m^2 s\,/\,rad)$

Spectrum

$\omega,\ (rad/s)$

Frequency

Fig. 1

Frequency spectrum evolution within time intervals:

a) $100 \leq t < 50000\ \sec$; b) $11000 \leq t < 15000\ \sec$; (----) – approximation $S \approx \omega^{-4.0}$

3.2 <u>Estimation of spectral front propagation</u>

In wave spectrum the evolution of two effects can be observed: a relatively slow growth of spectra in the forcing range and a fast "front propagation" to area of high wave numbers. It is a self-accelerating process. In paper (Komen et al., 1994), this concept is used to explain high frequency spectrum formation and the migration of the spectral peak towards lower frequencies.

A formation of the stationary spectra occures within a finite time interval. At the same time the wave energy propagation to high frequency range is observed. The speed of this propagation can be estimated as a shift of the spectral frequency front into high wave numbers (Fig. 2). The spectral frequency front is defined as a high frequency ("cut-off frequency"), with spectral density value being one order smaller in comparison with the limited spectrum solution for a given frequency within the range $W_f < W < W_p$. The same effect is described in paper (Donelan et al., 1985).
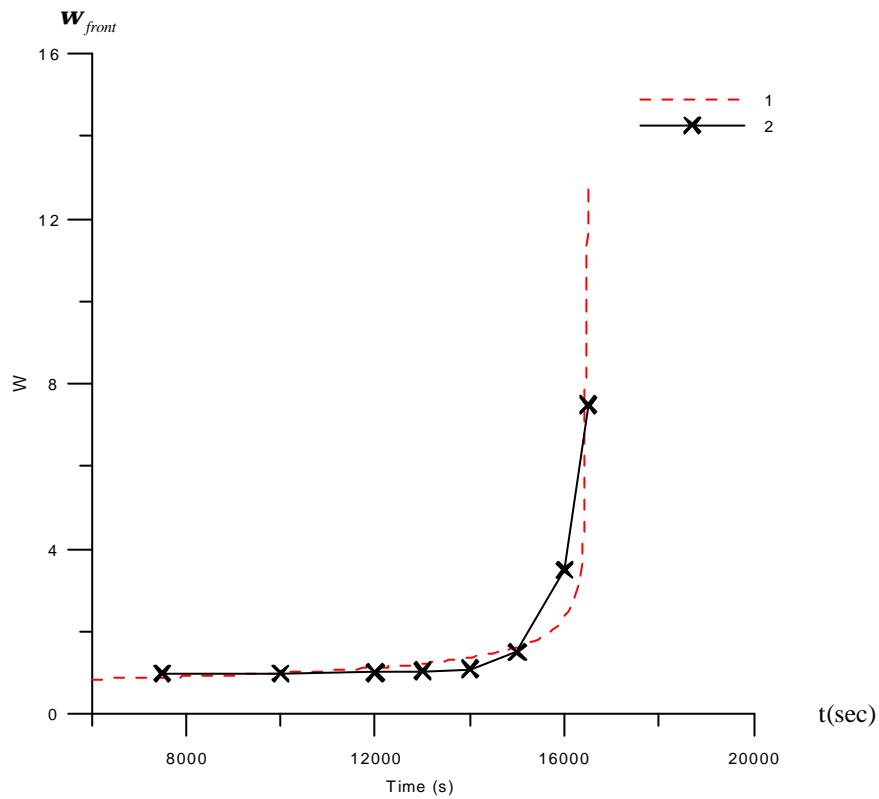

Fig.2

Propagation of spectral frequency front W_{front} into high frequency range

1 - data approximation (16), 2 – numerical data;

According to theoretical estimations (Zakharov et. al, 1992), the front propagation is approximated using a self-similar solution of the equation (1):

$$S(\omega, t) \cong (t_0 - t)^{4/3} R\left(\omega(t_0 - t)^{1/3}\right)$$

(15)

Hence, the "cut-off frequency" is increased explosively:

$$\omega_{front} \cong \frac{\omega_0}{(t_0 - t)^{1/3}} \tag{16}$$

Numerical estimations obtained in our experiment are in agreement with these theoretical results (Fig.2). A self-similar front propagating solution can be explained as a result of energy diffusion in frequency space in order to reach a system stable state.

4. ESTIMATION OF KOLMOGOROV'S CONSTANTS

4.1 Definition of Komlogorov's constants
According to the weak turbulence theory the general Kolmogorov spectrum is defined by fluxes of the wave energy P, wave action Q and momentum M (Zakharov et al., 1992). The spectrum is assumed to be symmetrical with respect to the reflection $\boldsymbol{b} \rightarrow -\boldsymbol{b}$. In a general case, the spectrum is as follows:

$$S(\boldsymbol{w}, \boldsymbol{b}) = \frac{g^{4/3} P^{1/3}}{\boldsymbol{w}^4} F\left(\frac{\boldsymbol{w} Q}{P}, \frac{g M}{\boldsymbol{w} P}, \cos \boldsymbol{b} \right) \tag{17}$$

In this paper only a case with no wave action flux at infinity: $Q = 0$ is considered. A similar solution was obtained analytically (Zakharov and Zaslavskii, 1982) for the infinite frequency interval, when energy pumping was located in the low frequency range and energy damping in the high frequency one.

For the large value $\boldsymbol{w}$, the function $F(\boldsymbol{x}, \boldsymbol{b})$ can be expanded into the Taylor series for small $\boldsymbol{x} = g M / \boldsymbol{w} P$:

$$F(\boldsymbol{x}, \boldsymbol{b}) \approx \boldsymbol{a}_0 + \boldsymbol{a}_1 \cos \boldsymbol{b} \tag{18}$$

For spectrum $S(\boldsymbol{w}, \boldsymbol{b})$, it can be presented as follows:

$$S(\boldsymbol{w}, \boldsymbol{b}) \cong \frac{g^{4/3} P^{1/3}}{\boldsymbol{w}^4} \left(\boldsymbol{a}_0 + \boldsymbol{a}_1 \frac{g M \cos(\boldsymbol{b})}{\boldsymbol{w} P} \right), \tag{19}$$

where α_0 and α_1 are the first and the second Kolmogorov constants, which are coefficients of the spectral density expansion:

$$\alpha_0 = \frac{1}{2\pi} \frac{\omega^4}{g^{4/3} P^{1/3}} \int_{-\pi}^{\pi} S(\omega, \beta) \, d\beta; \tag{20}$$

$$\alpha_1 = \frac{1}{\pi} \frac{\omega^5 P^{2/3}}{g^{7/3} M} \int_{-\pi}^{\pi} S(\omega, \beta) \cos(\beta) \, d\beta \tag{21}$$

The energy flux P, directed into the high frequency range, is estimated as:

$$P = \int_{-\pi}^{\pi} \int_{\omega_p}^{\infty} \gamma(\omega, \beta) S(\omega, \beta) \, d\omega \, d\beta \tag{22}$$

The momentum flux is estimated similarly:

$$M = \int_{-\pi}^{\pi} \int_{\omega_p}^{\infty} \gamma(\omega, \beta) S(\omega, \beta) \frac{k}{\omega} \cos \beta \, d\omega \, d\beta \tag{23}$$

4.2. Isotropic case

The value α_0 can be determined in isotropic case as follows:

$$\alpha_0 = \frac{S(\omega)\,\omega^4}{g^{4/3}\,P^{1/3}} \qquad (24)$$

According to the weak turbulence theory, the value α_0 is a constant, not depending on $\boldsymbol{w}$. As our estimates show:
$\alpha_0 = 0.303 \pm 0.033$ for time moment $t > 20000$ sec.

4.3. Non-isotropic case

Similar numerical simulations of the non-linear spectrum evolution are fulfilled for non-isotropic source function generating wave energy (12) with different values of the parameters n and A.
The time evolution of wave integral parameters (i.e. total energy, wave action and momentum) is shown in Fig.3. A full stabilization of these parameters is observed at $t \geq 10000$ sec.
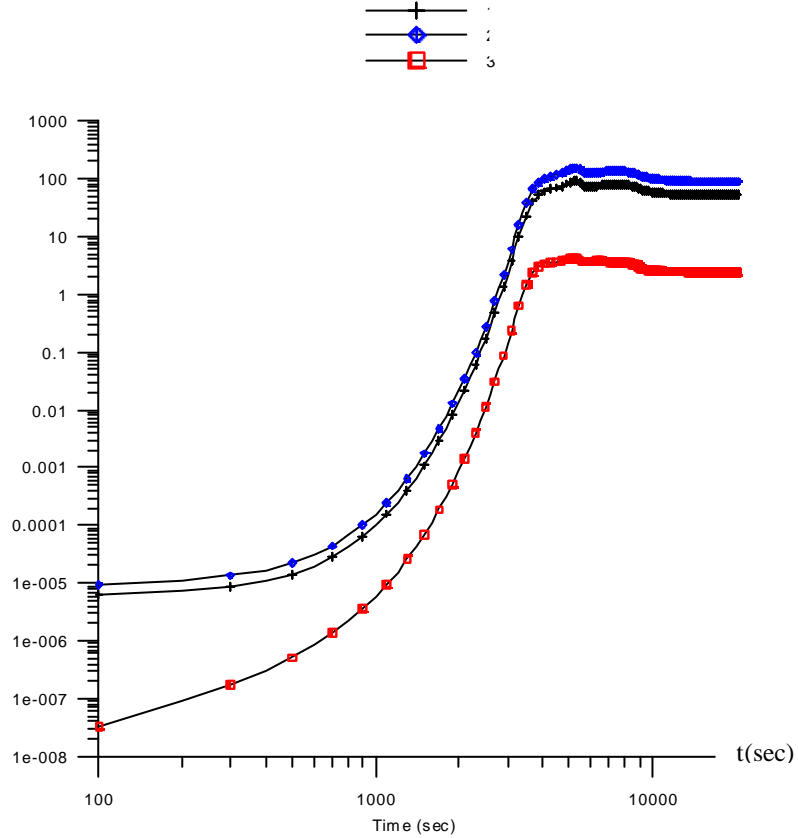

Fig.3
Time evolution of : 1- total energy, 2- wave action; 3 - momentum

Estimation results of the limited spectrum at $t=20000$ sec. for the most narrow angular distribution are presented in Fig.4 (frequency spectrum) and Fig.5 (frequency-angular spectrum). It should be pointed out that the main features of the frequency spectrum are nearly the same as in the isotropic case.
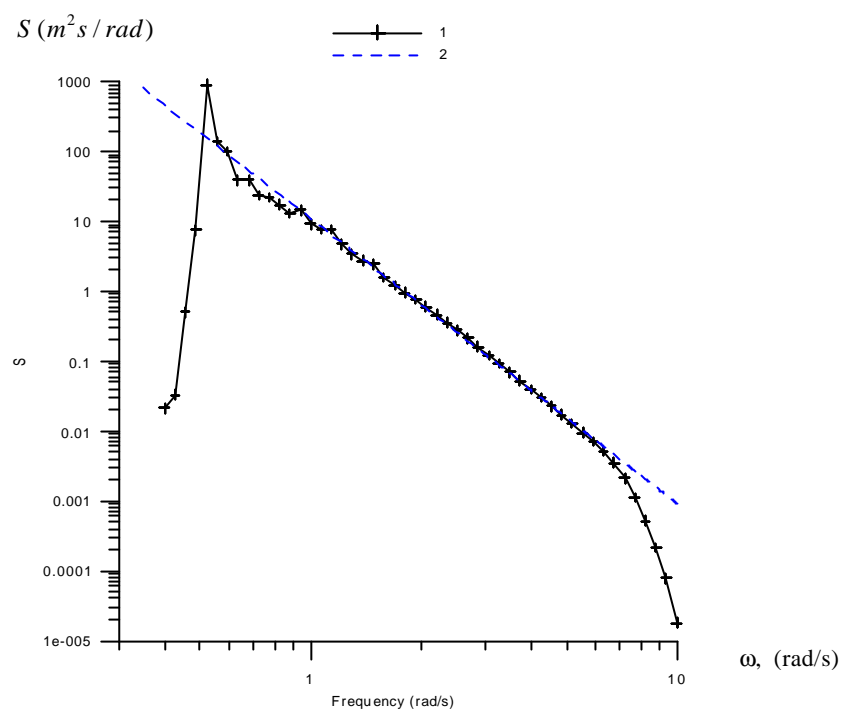
Fig. 4

Limited frequency spectrum in non-isotropic case , 1 – numerical data; 2- approximation $S \approx \omega^{-4.0}$.
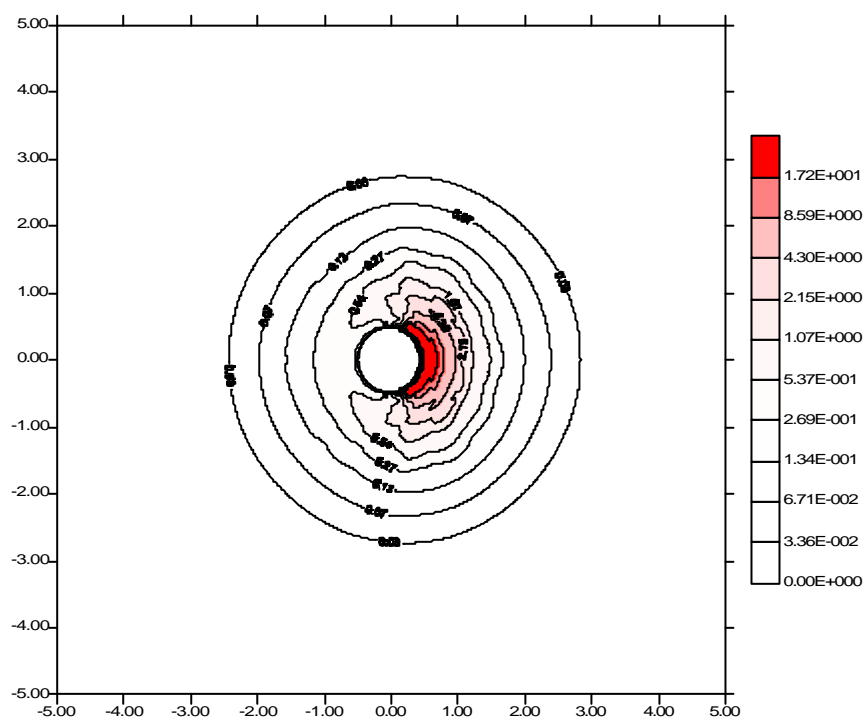

Fig.5

Limited frequency-angular spectrum

As for non-isotropic cases there is an additional possibility to estimate not only the first Kolmogorov constant, but also the second one using (21). The corresponding results are presented in Table 1

An angular spectrum width D (Fig.7) can be determined as a proportion of the frequency-angular spectrum value at the main direction to the frequency spectrum as follows:

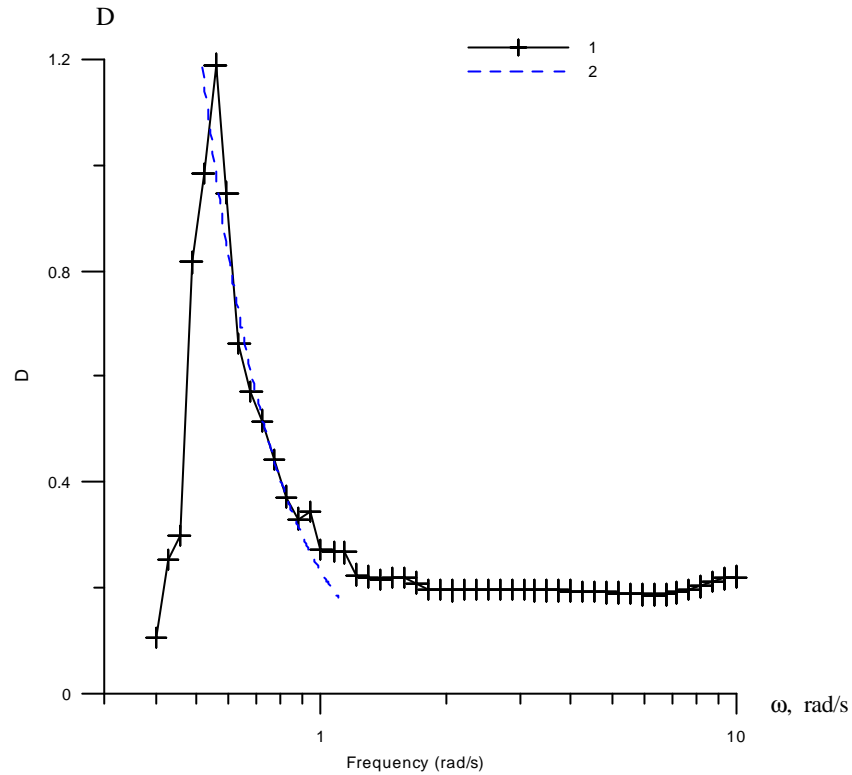$$D(\omega) = S(\omega, \beta_p)/S(\omega) \tag{25}$$

Fig. 6

Function $D(\omega)$

1- numerical results; 2 - local approximation: $D \approx 0.235 \cdot \omega^{-2.4}$

The function D is shown in Fig. 6. Its maximum value is achieved at the frequencies $\omega = 0.5 \div 0.6$ rad/s. After that it is quickly reduced as soon as the frequency is increased, showing that the function of angular energy distribution becomes wider.

Final estimation results of the Kolmogorov constants for different values of external angular functions are presented in Table 1. As it is seen the estimations of the first and the second constants are varied weakly depending on the external angular function distribution.

ESTIMATION OF KOLMOGOROV CONSTANTS FOR DIFFERENT ANGULAR DISTRIBUTION FUNCTIONS

TABLE 1

Angular function distribution of energy generating force	α_0	α_1
Isotropic	0.303 ± 0.033	
$\cos\ (\beta/2)$	0.308 ± 0.020	0.218 ± 0.015
$\cos^2(\beta)$	0.324 ± 0.021	0.239 ± 0.023
$\cos^4(\beta)$	0.311 ± 0.023	0.242 ± 0.034

It should be noted that the results presented in Table 1 are in general agreement with (Pushkarev et al., 2002), in which the following estimations are obtained: for isotropic case: $0.35 < a_0 < 0.45$ and for non-isotropic one: $0.33 < a_0 < 0.37$ and $0.18 < a_1 < 0.27$.

The discrepancies between the above mentioned results and ours can be explained as follows. In (Pushkarev et al., 2002) there are some differences between the values α_0 for isotropic and non-isotropic cases, whereas the values α_0 should be constant according to its definition. It should be noted that these estimations are obtained within a more wider confidence interval and only for two angular distributions. In our computation numerical simulations for four angular distributions are produced: from isotropic to a very narrow one typical for wind wave generation. Another reason of the discrepancies can be explained by the fact that there is no evidence that fully stabilized numerical solution is obtained. (Pushkarev et al., 2001). Moreover, our estimations can be considered to be more accurate due to the fact that a more precise numerical algorithm and more fine angular resolution are used.

5. COMPARISON WITH TOBA SPECTRUM

Toba (1973) made careful measurements of the spectrum tale and found out that the following approximation describes a spectrum:

$$S(\omega) = \delta \; g \, U_* \; \omega^{-4} \qquad (26)$$

where $\delta = 6.2 \cdot 10^{-2}$, U_* is a friction velocity .

It is interesting to make a comparison between the Toba spectrum and the Kolmogorov one (17) or (19). In order to do it the wave energy flux (22) should be defined. It includes the value of wave energy dissipation in a high frequency range, which is unknown. It can be estimated, supposing that the wind input energy flux is expended to support the high frequency energy flux directed to a high frequency range of wave spectrum:

$$P = \int\limits_{-\pi}^{\pi} \int\limits_{\omega_{min}}^{\omega_f} f(\omega, \beta) \, S(\omega, \beta) \, d\omega \, d\beta \qquad (27)$$

The wind wave input energy increment f can be estimated using a traditional approximation [Komen et al., 1994] . It is defined as follows:

$$f(\omega, \beta) = \max\left\{0; \; 0.25 \frac{\rho_a}{\rho_W} \omega \left(\frac{28 U_*}{c} \cos(\beta - \beta_U) - 1 \right) \right\}, \qquad (28)$$

where ρ_a and ρ_W is a density of air and water, correspondingly.

The angular-frequency spectrum approximation should be substituted into (22) in order to define the energy flux (22). Unfortunately an angular distribution of the Toba spectrum is unknown. Nevertheless it can be suggested that it is rather wide in high frequency range. The following estimation can be obtained for the isotropic frequency dependent spectrum:

$$P \cong \int\limits_{w_{min}}^{w_f} f(w) S(w) dw = 0.25 \cdot 10^{-3} dg U_* \int\limits_{w > g/28 U_*} w^{-3} \left(\frac{28 U_* w}{g} - 1 \right) dw \qquad (29)$$

Thus, the energy flux can be estimated as:

$$P \cong 0.125 \cdot 10^{-3} \delta g U_* \left(\frac{28 U_*}{g} \right)^2 \qquad (30)$$

Using (19) and (25), the first Kolmogorov constant can be estimated as follows:

$$\alpha_0 = \frac{S(\omega)\,\omega^4}{g^{4/3} P^{1/3}} = \frac{\delta g U_*}{0.1 g^{4/3} \sqrt[3]{98 \delta U_*^3 / g}} = 2.17 \delta^{2/3} = 0.33 \qquad (31)$$

Comparing this value with that obtained with the help of d irect numerical simulations (see Tabl. 1) a conclusion can be made that they are in a good agreement..

CONCLUSIONS

Direct numerical simulations of the Hasselmann kinetic equation for gravity waves in water surface confirms basic predictions of the weak-turbulent theory. The kinetic equation for surface gravity waves is investigated numerically taking into account an external generating force and dissipation. An efficient numerical algorithm for simulating non-linear energy transfer is used to solve the problem.

Three stages of wave development are revealed: unstable wave energy growth within a range of external force impact, fast energy spectrum tail formation in high frequency range and establishment of a steady state spectrum. In both isotropic and non-isotropic cases the spectra are found out to be dose to the Zakharov-Filonenko spectrum w^{-4} in the universal range. Reliable estimations of the Kolmogorov constants are found out. Formation of this asymptotic spectrum happens explosively. Accurate estimations of the first and second Kolmogorov constants are obtained.

A good agreement between the Toba experimental data and our results obtained with the help of direct numerical simulation is observed.

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